

eHDECAY: an Implementation of the Higgs Effective Lagrangian into HDECAY

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Abstract

We present **eHDECAY**, a modified version of the program **HDECAY** which includes the full list of leading bosonic operators of the Higgs effective Lagrangian with a linear or non-linear realization of the electroweak symmetry and implements two benchmark composite Higgs models.

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1 Introduction

In a companion paper [1], we gave a detailed review of the low-energy effective Lagrangian which describes a light Higgs-like boson and estimated the deviations induced by the leading operators to the Higgs decay rates. We discussed in particular how the effective Lagrangian can be used beyond the tree-level by performing a multiple perturbative expansion in the SM coupling parameter α/π and in powers of E/M , where E is the energy of the process and M is the New Physics (NP) scale at which new massive states appear. When the Higgs-like boson is part of a weak doublet, a third expansion must be performed for $v/f \ll 1$, where $f \equiv M/g_\star$ and g_\star is the typical coupling of the NP sector.

A recent study [2] concluded that, at tree-level, there are 8 dimension-6 CP-even operators that can be constrained by Higgs physics only. It is of course essential to have automatic tools to give accurate predictions of the deviations induced by these operators to Higgs observables. These operators are all part of the Strongly Interacting Light Higgs (SILH) Lagrangian [3] that we will be dealing with (the SILH Lagrangian, Eq. 2.2, contains 12 operators but 2 combinations of them are severely constrained by electroweak (EW) precision data and two other combinations are constrained by the bounds on anomalous triple gauge couplings). These operators are also included in Monte Carlo codes recently developed [4, 5].

The purpose of this note is to present the Fortran code `eHDECAY`, which implements the leading operators in the effective Lagrangian and gives an extension of the program `HDECAY` [6] for the automatic calculation of the Higgs decay widths and branching ratios. The program can be obtained at the URL:

<http://www-itp.particle.uni-karlsruhe.de/~maggie/eHDECAY/>.

The organization of the paper is as follows. In Section 2 we briefly review the definition of the effective Lagrangians, with linearly and non-linearly realized electroweak symmetry breaking (EWSB), that have been implemented in the program. This is mainly to set the notation. For more details and for a discussion of the physics implications we refer the reader to Ref. [1]. Section 3 gives a detailed discussion of how the partial decay widths have been implemented into the program `eHDECAY`, including higher-order effects in the perturbative expansion. For issues related to the perturbative expansion and the inclusion of higher-order corrections we again refer the reader to Ref. [1]. In Section 4 we give numerically

approximated results for the Higgs decay rates in the framework of linearly realized EWSB. Section 5 explains how to run `eHDECAY` and presents sample input and output files. We conclude in Section 6.

2 Effective Lagrangians for linearly and non-linearly realized EW symmetry

We assume for simplicity that the Higgs boson is CP-even and that baryon and lepton numbers are conserved. If the Higgs is part of a weak doublet, the leading effects beyond the Standard Model are parametrized by 53 operators with dimension-6 [7–9] (additional 6 operators must be added if the assumption of CP conservation is relaxed), when a single family of quarks and leptons is considered. In the following we will adopt the so-called SILH basis proposed in Ref. [3]:

$$\mathcal{L} = \mathcal{L}_{SM} + \sum_i \bar{c}_i O_i \equiv \mathcal{L}_{SM} + \Delta\mathcal{L}_{SILH} + \Delta\mathcal{L}_{F_1} + \Delta\mathcal{L}_{F_2} + \Delta\mathcal{L}_V + \Delta\mathcal{L}_{4F}, \quad (2.1)$$

with ²

$$\begin{aligned} \Delta\mathcal{L}_{SILH} = & \frac{\bar{c}_H}{2v^2} \partial^\mu (H^\dagger H) \partial_\mu (H^\dagger H) + \frac{\bar{c}_T}{2v^2} \left(H^\dagger \overleftrightarrow{D}^\mu H \right) \left(H^\dagger \overleftrightarrow{D}_\mu H \right) - \frac{\bar{c}_6 \lambda}{v^2} (H^\dagger H)^3 \\ & + \left(\left(\frac{\bar{c}_u}{v^2} y_u H^\dagger H \bar{q}_L H^c u_R + \frac{\bar{c}_d}{v^2} y_d H^\dagger H \bar{q}_L H d_R + \frac{\bar{c}_l}{v^2} y_l H^\dagger H \bar{L}_L H l_R \right) + h.c. \right) \\ & + \frac{i\bar{c}_W g}{2m_W^2} \left(H^\dagger \sigma^i \overleftrightarrow{D}^\mu H \right) (D^\nu W_{\mu\nu})^i + \frac{i\bar{c}_B g'}{2m_W^2} \left(H^\dagger \overleftrightarrow{D}^\mu H \right) (\partial^\nu B_{\mu\nu}) \\ & + \frac{i\bar{c}_{HW} g}{m_W^2} (D^\mu H)^\dagger \sigma^i (D^\nu H) W_{\mu\nu}^i + \frac{i\bar{c}_{HB} g'}{m_W^2} (D^\mu H)^\dagger (D^\nu H) B_{\mu\nu} \\ & + \frac{\bar{c}_\gamma g'^2}{m_W^2} H^\dagger H B_{\mu\nu} B^{\mu\nu} + \frac{\bar{c}_g g_S^2}{m_W^2} H^\dagger H G_{\mu\nu}^a G^{a\mu\nu}, \end{aligned} \quad (2.2)$$

²In this paper we follow the same notation as in Ref. [1]. In particular, the expression of the SM Lagrangian \mathcal{L}_{SM} and the convention for the covariant derivatives and the gauge field strengths can be found in Appendix A of Ref. [1].

$$\begin{aligned}
\Delta\mathcal{L}_{F_1} = & \frac{i\bar{c}_{Hq}}{v^2} (\bar{q}_L\gamma^\mu q_L) (H^\dagger \overleftrightarrow{D}_\mu H) + \frac{i\bar{c}'_{Hq}}{v^2} (\bar{q}_L\gamma^\mu \sigma^i q_L) (H^\dagger \sigma^i \overleftrightarrow{D}_\mu H) \\
& + \frac{i\bar{c}_{Hu}}{v^2} (\bar{u}_R\gamma^\mu u_R) (H^\dagger \overleftrightarrow{D}_\mu H) + \frac{i\bar{c}_{Hd}}{v^2} (\bar{d}_R\gamma^\mu d_R) (H^\dagger \overleftrightarrow{D}_\mu H) \\
& + \left(\frac{i\bar{c}_{Hud}}{v^2} (\bar{u}_R\gamma^\mu d_R) (H^{c\dagger} \overleftrightarrow{D}_\mu H) + h.c. \right) \\
& + \frac{i\bar{c}_{HL}}{v^2} (\bar{L}_L\gamma^\mu L_L) (H^\dagger \overleftrightarrow{D}_\mu H) + \frac{i\bar{c}'_{HL}}{v^2} (\bar{L}_L\gamma^\mu \sigma^i L_L) (H^\dagger \sigma^i \overleftrightarrow{D}_\mu H) \\
& + \frac{i\bar{c}_{Hl}}{v^2} (\bar{l}_R\gamma^\mu l_R) (H^\dagger \overleftrightarrow{D}_\mu H),
\end{aligned} \tag{2.3}$$

$$\begin{aligned}
\Delta\mathcal{L}_{F_2} = & \frac{\bar{c}_{uB} g'}{m_W^2} y_u \bar{q}_L H^c \sigma^{\mu\nu} u_R B_{\mu\nu} + \frac{\bar{c}_{uW} g}{m_W^2} y_u \bar{q}_L \sigma^i H^c \sigma^{\mu\nu} u_R W_{\mu\nu}^i + \frac{\bar{c}_{uG} g_S}{m_W^2} y_u \bar{q}_L H^c \sigma^{\mu\nu} \lambda^a u_R G_{\mu\nu}^a \\
& + \frac{\bar{c}_{dB} g'}{m_W^2} y_d \bar{q}_L H \sigma^{\mu\nu} d_R B_{\mu\nu} + \frac{\bar{c}_{dW} g}{m_W^2} y_d \bar{q}_L \sigma^i H \sigma^{\mu\nu} d_R W_{\mu\nu}^i + \frac{\bar{c}_{dG} g_S}{m_W^2} y_d \bar{q}_L H \sigma^{\mu\nu} \lambda^a d_R G_{\mu\nu}^a \\
& + \frac{\bar{c}_{lB} g'}{m_W^2} y_l \bar{L}_L H \sigma^{\mu\nu} l_R B_{\mu\nu} + \frac{\bar{c}_{lW} g}{m_W^2} y_l \bar{L}_L \sigma^i H \sigma^{\mu\nu} l_R W_{\mu\nu}^i + h.c..
\end{aligned} \tag{2.4}$$

Here λ denotes the Higgs quartic coupling which appears in the SM Lagrangian \mathcal{L}_{SM} , and the weak scale is defined by

$$v \equiv \frac{1}{(\sqrt{2}G_F)^{1/2}} \simeq 246 \text{ GeV}. \tag{2.5}$$

We have defined the Hermitian derivative

$$iH^\dagger \overleftrightarrow{D}^\mu H \equiv iH^\dagger (D^\mu H) - i(D^\mu H)^\dagger H \tag{2.6}$$

and $\sigma^{\mu\nu} \equiv i[\gamma^\mu, \gamma^\nu]/2$. The Yukawa couplings $y_{u,d,l}$ and the Wilson coefficients \bar{c}_i are matrices in flavor space, and a summation over flavor indices has been implicitly assumed. In order to avoid large Flavor-Changing Neutral Currents (FCNC) through the tree-level exchange of the Higgs boson, we assume that each of the operators $O_{u,d,l}$ is flavor-aligned with the corresponding mass term. The coefficients $\bar{c}_{u,d,l}$ are then proportional to the identity matrix in flavor space. Furthermore, as we assume CP-invariance, they are taken to be real. A naive estimate of the Wilson coefficients \bar{c}_i can be found in Eq. (2.9) of Ref. [1], following the power counting of Ref. [3].

In addition to those listed in Eqs. (2.2)-(2.4), the effective Lagrangian includes also five extra bosonic operators, $\Delta\mathcal{L}_V$, as well as 22 four-Fermi baryon-number conserving operators, $\Delta\mathcal{L}_{4F}$. Two of the operators in Eqs. (2.2), (2.3) are in fact redundant and can be eliminated through the equations of motion. A most convenient choice is that of eliminating two of the three operators involving leptons in $\Delta\mathcal{L}_{F_1}$.

In the unitary gauge with canonically normalized fields, the SILH effective Lagrangian $\Delta\mathcal{L}_{SILH}$ reads:

$$\begin{aligned}
\mathcal{L} = & \frac{1}{2}\partial_\mu h \partial^\mu h - \frac{1}{2}m_h^2 h^2 - c_3 \frac{1}{6} \left(\frac{3m_h^2}{v} \right) h^3 - \sum_{\psi=u,d,l} m_{\psi^{(i)}} \bar{\psi}^{(i)} \psi^{(i)} \left(1 + c_\psi \frac{h}{v} + \dots \right) \\
& + m_W^2 W_\mu^+ W^{-\mu} \left(1 + 2c_W \frac{h}{v} + \dots \right) + \frac{1}{2}m_Z^2 Z_\mu Z^\mu \left(1 + 2c_Z \frac{h}{v} + \dots \right) + \dots \\
& + \left(c_{WW} W_{\mu\nu}^+ W^{-\mu\nu} + \frac{c_{ZZ}}{2} Z_{\mu\nu} Z^{\mu\nu} + c_{Z\gamma} Z_{\mu\nu} \gamma^{\mu\nu} + \frac{c_{\gamma\gamma}}{2} \gamma_{\mu\nu} \gamma^{\mu\nu} + \frac{c_{gg}}{2} G_{\mu\nu}^a G^{a\mu\nu} \right) \frac{h}{v} \\
& + \left((c_{W\partial W} W_\nu^- D_\mu W^{+\mu\nu} + h.c.) + c_{Z\partial Z} Z_\nu \partial_\mu Z^{\mu\nu} + c_{Z\partial\gamma} Z_\nu \partial_\mu \gamma^{\mu\nu} \right) \frac{h}{v} + \dots
\end{aligned} \tag{2.7}$$

where we have shown terms with up to three fields and at least one Higgs boson. The couplings c_i are linear functions of the Wilson coefficients of the effective Lagrangian (2.1) and are reported in Table 1.³ In particular, the following relations hold

$$c_{WW} - c_{ZZ} \cos^2 \theta_W = c_{Z\gamma} \sin 2\theta_W + c_{\gamma\gamma} \sin^2 \theta_W \tag{2.8}$$

$$c_{W\partial W} - c_{Z\partial Z} \cos^2 \theta_W = \frac{c_{Z\partial\gamma}}{2} \sin 2\theta_W, \tag{2.9}$$

which are a consequence of the accidental custodial invariance of the SILH Lagrangian at the level of dimension-6 operators [1].⁴ Imposing custodial invariance for the Lagrangian (2.2), so that $\bar{c}_T = 0$, implies a third relation that holds for the non-derivative couplings c_W and c_Z :

$$c_W = c_Z. \tag{2.10}$$

For arbitrary values of the couplings c_i , Eq. (2.7) represents the most general effective Lagrangian which can be written at $O(p^4)$ in a derivative expansion by focusing on cubic terms

³Notice that the similar Table 1 in Ref. [1] contains an erroneous factor 2 in the dependence of c_Z on \bar{c}_T .

⁴If the assumption of CP conservation is relaxed, $c_{W\partial W}$ can in general be complex, while the other bosonic couplings of Eq. (2.7) are real. In this case Eq. (2.9) corresponds to two real identities, respectively, on the real and on the imaginary parts, so that custodial symmetry implies $\text{Im}(c_{W\partial W}) = 0$.

Higgs couplings	$\Delta\mathcal{L}_{SILH}$	MCHM4	MCHM5
c_W	$1 - \bar{c}_H/2$	$\sqrt{1-\xi}$	$\sqrt{1-\xi}$
c_Z	$1 - \bar{c}_H/2 - \bar{c}_T$	$\sqrt{1-\xi}$	$\sqrt{1-\xi}$
c_ψ ($\psi = u, d, l$)	$1 - (\bar{c}_H/2 + \bar{c}_\psi)$	$\sqrt{1-\xi}$	$\frac{1-2\xi}{\sqrt{1-\xi}}$
c_3	$1 + \bar{c}_6 - 3\bar{c}_H/2$	$\sqrt{1-\xi}$	$\frac{1-2\xi}{\sqrt{1-\xi}}$
c_{gg}	$8(\alpha_s/\alpha_2)\bar{c}_g$	0	0
$c_{\gamma\gamma}$	$8\sin^2\theta_W\bar{c}_\gamma$	0	0
$c_{Z\gamma}$	$(\bar{c}_{HB} - \bar{c}_{HW} - 8\bar{c}_\gamma\sin^2\theta_W)\tan\theta_W$	0	0
c_{WW}	$-2\bar{c}_{HW}$	0	0
c_{ZZ}	$-2(\bar{c}_{HW} + \bar{c}_{HB}\tan^2\theta_W - 4\bar{c}_\gamma\tan^2\theta_W\sin^2\theta_W)$	0	0
$c_{W\partial W}$	$-2(\bar{c}_W + \bar{c}_{HW})$	0	0
$c_{Z\partial Z}$	$-2(\bar{c}_W + \bar{c}_{HW}) - 2(\bar{c}_B + \bar{c}_{HB})\tan^2\theta_W$	0	0
$c_{Z\partial\gamma}$	$2(\bar{c}_B + \bar{c}_{HB} - \bar{c}_W - \bar{c}_{HW})\tan\theta_W$	0	0

Table 1: The second column reports the values of the Higgs couplings c_i defined in Eq. (2.7) in terms of the coefficients \bar{c}_i of the effective Lagrangian $\Delta\mathcal{L}_{SILH}$. The last two columns show the predictions of the MCHM4 and MCHM5 models in terms of $\xi = (v/f)^2$, see Ref. [1] for details. The auxiliary parameter α_2 is defined by Eq. (3.12).

with at least one Higgs boson and making the following two assumptions: *i*) CP is conserved; *ii*) vector fields couple to conserved currents. Effects which violate the second assumption, in particular, are suppressed by the fermion masses, hence they are small for all the processes of interest in this work. Such description does not require the Higgs boson to be part of an electroweak doublet, and in this sense Eq. (2.7) can be considered as a generalization of the SILH Lagrangian $\Delta\mathcal{L}_{\text{SILH}}$. It contains 10 couplings involving a single Higgs boson and two gauge fields (hVV couplings, with $V = W, Z, \gamma, g$), 3 linear combinations of which vanish if custodial symmetry is imposed [1]. This counting agrees with the complete non-linear Lagrangian at $\mathcal{O}(p^4)$ recently built in Refs. [10–13]. This general Lagrangian contains many more operators but it can be easily checked that only 10 independent operators remain after assuming CP invariance and the conservation of fermionic currents, and among them 3 break the custodial symmetry. If the assumption on conserved currents is relaxed, there are two more independent operators at $\mathcal{O}(p^4)$ that give rise to hVV couplings (they are the operators P_9 and P_{10} of Ref. [13], see also the general form factor description of Ref. [14]). These two additional couplings can only be obtained from dimension-8 operators when the Higgs boson is part of an EW doublet. In the non-linear realization of the EW symmetry, all Higgs couplings are truly independent of other parameters that do not involve the Higgs boson, like EW oblique parameters or anomalous triple gauge couplings. In a linear realization, on the other hand, only 4 hVV couplings are independent of the other EW measurements [2]. In custodial invariant scenarios, it is thus not possible to tell whether the Higgs is part of an EW doublet by focusing only on hVV couplings, since their number is the same in both the linear and non-linear descriptions under our assumptions (CP and current conservation). The decorrelation between the hVV couplings and the other EW data might instead be a way to disprove the doublet nature of the Higgs boson [13].

The code `eHDECAY` retains only the couplings induced by the operators of $\Delta\mathcal{L}_{\text{SILH}}$ since the effects of the other operators with fermions are either severely constrained by non-Higgs physics or, like the top dipoles, are irrelevant for the Higgs total decay rates (they could modify in a sensible way the differential decay rates but such an analysis is beyond the scope of the present work). The CP-odd operators are not considered either since they do not interfere with the inclusive SM amplitudes and thus modify the decay rates at a subleading order in the perturbative expansion considered in this paper.

3 Implementation of the Higgs effective Lagrangian into eHDECAY

The program HDECAY [6] was originally written for the automatic computation of the Higgs partial decay widths and branching ratios in the SM and in its Minimal Supersymmetric extension (MSSM). It includes the possibility of specifying modified couplings for up-type quarks, down-type quarks, leptons and vector bosons in the parametrization of Eq. (2.7), as well as of including the effective couplings c_{gg} , $c_{\gamma\gamma}$ and $c_{Z\gamma}$. We present here a modified version of the program, labelled **eHDECAY**. It is available at the following URL:

<http://www-itp.particle.uni-karlsruhe.de/~maggie/eHDECAY/>.

In addition to the features already present in HDECAY, the new program includes the effective couplings c_{WW} , c_{ZZ} , $c_{W\partial W}$ and $c_{Z\partial Z}$, and thus fully implements the non-linear Lagrangian (2.7).⁵ In fact, similarly to HDECAY 5.10, it also includes the possibility of choosing different couplings of the Higgs boson to each of the up and down quark flavors and lepton flavors. In this sense the program assumes neither custodial symmetry nor flavor alignment. As explained in the text, Eq. (2.7) describes a generic CP-even scalar h at $O(p^4)$ in the derivative expansion. If h forms an $SU(2)_L$ doublet together with the longitudinal polarizations of the W and the Z , the Lagrangian can be expanded as in Eq. (2.2) for $(v/f) \ll 1$; in this case the values of the Higgs couplings c_i are given in the second column of Table 1. The program eHDECAY provides an option in its input file where the user can switch from the non-linear parametrization of Eq. (2.7) to that of the SILH Lagrangian Eq. (2.2). The user can also choose to set the values of the Higgs couplings to those predicted at leading order in an expansion in powers of weak couplings in the benchmark composite Higgs models MCHM4 [15] and MCHM5 [16], see the last two columns of Table 1.

Similarly to the original version of HDECAY, all the relevant QCD corrections are included. They generally factorize with respect to the expansion in the number of fields and derivatives of the effective Lagrangian, and can thus be straightforwardly included by making use of the existing SM computations. The inclusion of the electroweak corrections is less straightforward and can currently be done in a consistent way only in the framework of the

⁵Notice that the operator proportional to $c_{Z\partial\gamma}$ does not affect the decay $h \rightarrow Z\gamma$ as long as the photon is on-shell.

Lagrangian (2.2) and up to higher orders in (v/f) . Going beyond such approximations would require dedicated computations which at the moment are not available in the literature. In **eHDECAY** the user has the option to include the one-loop EW corrections to a given decay rate only if the parametrization of Eq. (2.2) has been chosen. The same EW scheme as used by **HDECAY**, with G_F , m_W and m_Z taken as input parameters, is also adopted in **eHDECAY**. The sine of the Weinberg angle is defined as

$$\sin^2\theta_W = 1 - \frac{m_W^2}{m_Z^2}, \quad (3.11)$$

following the conventional on-shell scheme [17]. Derived quantities in this scheme are also the electromagnetic coupling and the weak coupling. To describe the latter we have conveniently defined the parameter

$$\alpha_2 \equiv \frac{\sqrt{2}G_F m_W^2}{\pi}. \quad (3.12)$$

The formulas implemented in the program are thus written in terms of only the input parameters or their derived quantities $\sin\theta_W$ and α_2 . The only exception to this rule is given by the decay rates $\Gamma(h \rightarrow \gamma\gamma)$ and $\Gamma(h \rightarrow Z\gamma)$, where we use the experimental value of the electromagnetic coupling in the Thomson limit, $\alpha_{em}(q^2 = 0)$, in order to avoid large logarithms for on-shell photons.

Below a detailed discussion follows of how the New Physics corrections are incorporated for each of the Higgs decay modes. We report explicitly the formulas implemented in the code and their level of approximation in the perturbative expansion of the effective Lagrangian. In all the following expressions, as explained in the text, the coefficients of the dimension-6 operators of the SILH Lagrangian (2.2) and those of the derivative operators of Eq. (2.7) must be identified with their values at the relevant low-energy scale $\mu = m_h$.

3.1 Decays into quarks and leptons

Upon adopting the effective description of the non-linear Lagrangian (2.7) and working at leading order in the derivative expansion, the Higgs boson partial decay width into a pair of fermions is obtained by rescaling the tree-level SM value $\Gamma_0^{SM}(\psi\bar{\psi})$ by a factor c_ψ^2 . The QCD corrections to the decay widths into quarks which are currently available for the SM case include fully massive next-to-leading order (NLO) corrections near threshold [18]

and massless $\mathcal{O}(\alpha_s^4)$ corrections far above threshold [19–22]. Also, large logarithms can be resummed through the running of the quark masses and of the strong coupling constant. They are evaluated at the scale given by the Higgs mass. The transition from the threshold region involving mass effects to the renormalization-group-improved large-Higgs mass regime is provided by a smooth linear interpolation. All these QCD corrections factorize with respect to the tree-level amplitude and can therefore be incorporated as done in **HDECAY** for the SM case. The decay rate can be written as follows:

$$\Gamma(\bar{\psi}\psi)|_{NL} = c_\psi^2 \Gamma_0^{SM}(\bar{\psi}\psi) [1 + \delta_\psi \kappa^{QCD}] , \quad (3.13)$$

where Γ_0^{SM} denotes the leading-order decay width, $\delta_\psi = 1(0)$ for $\psi = \text{quark (lepton)}$ and κ^{QCD} encodes the QCD corrections.⁶ This is the formula implemented by **eHDECAY** in the case of the non-linear Lagrangian (2.7). It is valid up to corrections of $O(m_h^2/M^2)$ in the derivative expansion and of $O(\alpha_2/4\pi)$ from EW loops. These latter corrections are available in the SM but contrary to the QCD ones do not factorize. Their inclusion in the case of generic Higgs couplings thus requires a dedicated calculation, which is not available at present. The two benchmark composite Higgs models MCHM4 and MCHM5 provide a resummation of higher-order terms in $\xi = v^2/f^2$. Contrary to the SILH Lagrangian which is to be seen as an expansion in ξ , in these two models rather large coupling deviations can in principle be possible (eventually they are precluded due to the constraints from electroweak precision measurements). We therefore apply the formula Eq. (3.13) also for the MCHM4 and MCHM5, with c_ψ given by the corresponding coupling values in columns 3 and 4 of Table 1.

In case of the SILH parametrization, where the deviations of the Higgs couplings from their SM values are assumed to be of $O(v^2/f^2)$ and small, the decay rate can be written as

$$\Gamma(\bar{\psi}\psi)|_{SILH} = \Gamma_0^{SM}(\bar{\psi}\psi) \left[1 - \bar{c}_H - 2\bar{c}_\psi + \frac{2}{|A_0^{SM}|^2} \text{Re}(A_0^{*SM} A_{1,ew}^{SM}) \right] [1 + \delta_\psi \kappa^{QCD}] , \quad (3.14)$$

where A_0^{SM} , $A_{1,ew}^{SM}$ are, respectively, the tree-level and EW one-loop [23] amplitudes of the SM. In this case the one-loop EW corrections can be easily included if one neglects

⁶There is one caveat, however. In the case of decays into strange, charm or bottom quarks there are two-loop diagrams which involve loops of top quarks coupling to the Higgs boson. They need a rescaling different from c_ψ^2 . It has been correctly taken into account by the appropriate modification factor $c_\psi c_t$ ($\psi = c, s, b$).

terms of $O[(\alpha_2/4\pi)(v/f)^2]$ ⁷. In particular, mixed contributions up to $O[(\alpha_2/4\pi)(\alpha_s/4\pi)^4]$ have been included by assuming that the electroweak and QCD corrections factorize, as the non-factorizable contributions are small. From the viewpoint of the expansion in inverse powers of the NP scale, the formula (3.14) includes corrections of order $O(v^2/f^2)$. It neglects terms of $O(v^4/f^4)$, $O[(\alpha_2/4\pi)(v/f)^2]$, $O[(\alpha_2/4\pi)^2]$.

3.2 Decay into gluons

Upon selecting the Lagrangian (2.7), the rate into two gluons is computed in **eHDECAY** by means of the following formula:

$$\begin{aligned} \Gamma(gg)|_{NL} = & \frac{G_F \alpha_s^2 m_h^3}{4\sqrt{2}\pi^3} \left[\left| \sum_{q=t,b,c} \frac{c_q}{3} A_{1/2}(\tau_q) \right|^2 c_{eff}^2 \kappa_{soft} \right. \\ & + 2 \operatorname{Re} \left(\sum_{q=t,b,c} \frac{c_q}{3} A_{1/2}^*(\tau_q) \frac{2\pi c_{gg}}{\alpha_s} \right) c_{eff} \kappa_{soft} + \left| \frac{2\pi c_{gg}}{\alpha_s} \right|^2 \kappa_{soft} \quad (3.15) \\ & \left. + \frac{1}{9} \sum_{q,q'=t,b} c_q A_{1/2}^*(\tau_q) c_{q'} A_{1/2}(\tau_{q'}) \kappa^{NLO}(\tau_q, \tau_{q'}) \right], \end{aligned}$$

where $\tau_q = 4m_q^2/m_h^2$ and the loop function, normalized to $A_{1/2}(\infty) = 1$, is defined as

$$A_{1/2}(\tau) = \frac{3}{2} \tau [1 + (1 - \tau) f(\tau)], \quad (3.16)$$

with

$$f(\tau) = \begin{cases} \arcsin^2 \frac{1}{\sqrt{\tau}} & \tau \geq 1 \\ -\frac{1}{4} \left[\ln \frac{1 + \sqrt{1 - \tau}}{1 - \sqrt{1 - \tau}} - i\pi \right]^2 & \tau < 1. \end{cases} \quad (3.17)$$

The first term corresponds to the one-loop contribution from the top, bottom and charm quarks, whose couplings to the Higgs boson are modified with respect to their SM values.

⁷As pointed out in footnote 21 of Ref. [1], in the strict sense this equation is valid for the genuine EW corrections only, while for simplicity we include the (IR-divergent) virtual QED corrections to the SM amplitude in the same way. The corresponding real photon radiation contributions to the decay rates are treated in terms of a *linear* novel contribution to the Higgs coupling for the squared amplitude in order to obtain an infrared finite result. Pure QED corrections factorize as QCD corrections in general so that their amplitudes scale with the modified Higgs couplings. However, they cannot be separated from the genuine EW corrections in a simple way.

In order to minimize the effects from higher-order QCD corrections, we use the pole masses for the top, bottom and charm quarks, $m_t = 172.5 \text{ GeV}$, $m_b = 4.75 \text{ GeV}$ and $m_c = 1.42 \text{ GeV}$. The second and third terms encode the effect of the derivative interaction between the Higgs boson and two gluons generated by New Physics. Naively $c_{gg} \approx (\alpha_s/4\pi)(g_*^2 v^2/M^2)$, so that the correction from the effective interaction can be as important as the one from the top quark if $(g_*^2 v^2/M^2) \approx 1$. No expansion is thus possible in c_{gg} in the general case.

The QCD corrections have been included up to N³LO in Eq. (3.15) in the limit of large loop-particle masses, similarly to what is done in **HDECAY** for the SM. In this limit the effect of soft radiation factorizes and is encoded by the coefficient κ_{soft} . The coefficient c_{eff} , instead, takes into account the correction from the exchange of hard gluons and quarks with virtuality $q^2 \gg m_t^2$. More in detail, for $m_h \ll 2m_t$, one can integrate out the top quark and obtain the following five-flavour effective Lagrangian

$$\mathcal{L}_{\text{eff}} = -2^{1/4} G_F^{1/2} C_1 G_{a\mu\nu}^0 G_a^{0\mu\nu} h, \quad (3.18)$$

where bare fields are labeled by the superscript 0. The renormalized coefficient function C_1 encodes the dependence on the top quark mass m_t . The coefficients κ_{soft} and c_{eff} are thus defined as

$$\begin{aligned} \kappa_{soft} &= \frac{\pi}{2m_h^4} \text{Im } \Pi^{GG}(q^2 = m_h^2) \\ c_{eff} &= -\frac{12\pi C_1}{\alpha_s^{(5)}(m_h)}, \end{aligned} \quad (3.19)$$

where $\Pi^{GG}(q^2)$ is the vacuum polarization induced by the gluon operator. The N³LO expression of the coefficient function C_1 [24–27] in the on-shell scheme and that of $\text{Im } \Pi^{GG}$ can be found in Ref. [28]. At NLO the expressions for κ_{soft} and c_{eff} take the well-known form [29]

$$\kappa_{soft}^{NLO} = 1 + \frac{\alpha_s}{\pi} \left(\frac{73}{4} - \frac{7}{6} N_F \right), \quad c_{eff}^{NLO} = 1 + \frac{\alpha_s}{\pi} \frac{11}{4}, \quad (3.20)$$

where here α_s is evaluated at the scale m_h and computed for $N_F = 5$ active flavours. In **eHDECAY** it is consistently computed up to N³LO. The last line in Eq. (3.15) contains the additional mass effects at NLO QCD [30] in the top and bottom loops, encoded in $\kappa^{NLO}(\tau_q, \tau_{q'})$, which have been explicitly implemented in **HDECAY** and taken over in **eHDECAY**. While the mass effects for the top quark loops play only a minor role, below the percent

level, for the bottom loop contribution the mass effects for a 125 GeV Higgs boson amount to about 8% relative to the approximate NLO result. Hence, formula (3.15) includes the QCD corrections at N³LO (i.e. at $O(\alpha_s^5)$ in the decay rate), and neglects next-to-leading order terms in the derivative expansion (i.e. terms further suppressed by $O(m_h^2/M^2)$). The decay width within the MCHM4 and MCHM5 is calculated with the same formula (3.15) by replacing c_q with the values in column 3 and 4 of Table 1 and $c_{gg} \equiv 0$.

When the SILH Lagrangian (2.2) is selected, on the other hand, **eHDECAY** computes the decay rate into gluons by means of the following approximate formula:

$$\begin{aligned} \Gamma(gg)|_{SILH} = \frac{G_F \alpha_s^2 m_h^3}{4\sqrt{2}\pi^3} & \left[\frac{1}{9} \sum_{q,q'=t,b,c} (1 - \bar{c}_H - \bar{c}_q - \bar{c}_{q'}) A_{1/2}^*(\tau_{q'}) A_{1/2}(\tau_q) c_{eff}^2 \kappa_{soft} \right. \\ & + 2 \operatorname{Re} \left(\sum_{q=t,b,c} \frac{1}{3} A_{1/2}^*(\tau_q) \frac{16\pi \bar{c}_g}{\alpha_2} \right) c_{eff} \kappa_{soft} \\ & + \left| \sum_{q=t,b,c} \frac{1}{3} A_{1/2}(\tau_q) \right|^2 c_{eff}^2 \kappa_{ew} \kappa_{soft} \\ & \left. + \frac{1}{9} \sum_{q,q'=t,b} (1 - \bar{c}_H - \bar{c}_q - \bar{c}_{q'}) A_{1/2}^*(\tau_q) A_{1/2}(\tau_{q'}) \kappa^{NLO}(\tau_q, \tau_{q'}) \right]. \end{aligned} \quad (3.21)$$

The last line contains the mass effects at NLO QCD for the top and bottom quark loops. The NLO electroweak corrections [31, 32] are included through the coefficient κ_{ew} and by neglecting terms of $O[(\alpha_2/4\pi)(v^2/f^2)]$. The above formula thus includes the leading $O(v^2/f^2)$ corrections, as well as mixed $O[(\alpha_s/4\pi)^5(\alpha_2/4\pi)]$ ones. Indeed, we assume factorization of the QCD and EW corrections. Since QCD corrections are dominated by soft gluon radiation, in which QCD and EW effects completely factorize, this is a good approximation⁸. It neglects terms of $O[(\alpha_2/4\pi)^2]$ and $O(v^4/f^4)$.

3.3 Decay into photons

In the SM the decay of the Higgs boson into a pair of photons is mediated by W and heavy fermion loops. According to the chiral Lagrangian (2.7), these two contributions

⁸Bottom loops contribute $O(10\%)$ to the SM decay rate and are well approximated by an effective coupling at the 10%-level thus leading to negligible non-factorizing contributions at the percent level.

to the total amplitude are rescaled, respectively, by the parameters c_W and c_ψ . Similarly to $h \rightarrow gg$, the contact interaction proportional to $c_{\gamma\gamma}$ can also contribute significantly. With $c_{\gamma\gamma} \approx (\alpha_{em}/4\pi)(g_*^2 v^2/M^2)$, the contribution due to the effective interaction becomes comparable to the loop induced contributions if $(g_*^2 v^2/M^2) \approx 1$. The partial width for a Higgs boson decaying into two photons implemented in **eHDECAY** in the framework of the non-linear Lagrangian is thus given by

$$\Gamma(\gamma\gamma)|_{NL} = \frac{G_F \alpha_{em}^2 m_h^3}{128 \sqrt{2} \pi^3} \left| \sum_{q=t,b,c} \frac{4}{3} c_q 3Q_q^2 A_{1/2}^{NLO}(\tau_q) + \frac{4}{3} c_\tau Q_\tau^2 A_{1/2}(\tau_\tau) + c_W A_1(\tau_W) + \frac{4\pi}{\alpha_{em}} c_{\gamma\gamma} \right|^2, \quad (3.22)$$

which is approximate at leading order in the derivative expansion, i.e. it neglects terms further suppressed by $O(m_h^2/M^2)$. By $Q_{q,\tau}$ we denote the electric charge of the quarks and the τ lepton, respectively. Note that α_{em} is the electromagnetic coupling in the Thomson limit, in order to avoid large logarithms for on-shell photons. We have defined $\tau_i = 4m_i^2/m_h^2$ ($i = q, \tau, W$) and the form factor

$$A_1(\tau) = -[2 + 3\tau + 3\tau(2 - \tau)f(\tau)] \quad (3.23)$$

normalized to $A_1(\infty) = -7$. The top, bottom and charm quark loops receive NLO QCD corrections, while the effective contact interaction does not. The NLO QCD corrected quark form factor is denoted in Eq. (3.22) by

$$A_{1/2}^{NLO}(\tau_q) = A_{1/2}(\tau_q)(1 + \kappa_{QCD}), \quad (3.24)$$

where κ_{QCD} encodes the $O(\alpha_s/4\pi)$ QCD corrections [30, 33, 34] and $A_{1/2}(\tau)$ is given in Eq. (3.16). In the MCHM4 and MCHM5 we use the same formula for the decay width with c_q and c_V replaced appropriately and $c_{\gamma\gamma} \equiv 0$.

In order to improve the perturbative behaviour of the QCD-corrected quark loop contributions, they are expressed in terms of the running quark masses $m_Q(\mu_Q^2)$ [30, 33]. These are related to the pole masses M_Q through

$$m_Q(\mu_Q^2) = M_Q \left[\frac{\alpha_s(\mu_Q^2)}{\alpha_s(M_Q^2)} \right]^{12/(33-2N_F)} (1 + \mathcal{O}(\alpha_s^2)) \quad (3.25)$$

at the mass renormalization point μ_Q with $N_F = 5$ active flavours. Their scale is identified with $\mu_Q = M_H/2$. This ensures a proper definition of the $Q\bar{Q}$ thresholds $M_H = 2M_Q$ without artificial displacements due to finite shifts between the pole and the running quark masses, as is the case for the running $\overline{\text{MS}}$ masses. Note, that the same running quark mass $m_Q(\mu_Q^2)$, at the renormalization scale $\mu_Q = M_H/2$, enters in the lowest order amplitude $A_{1/2}^{LO}$, which is used in the SILH parametrization hereafter.⁹

In the case of the SILH parametrization, the EW corrections have been incorporated as well. It is useful to define the SM amplitude at leading order (LO) and NLO QCD level as

$$A_X^{SM}(\gamma\gamma) = \sum_{q=t,b,c} \frac{4}{3} 3Q_q^2 A_{1/2}^X(\tau_q) + \frac{4}{3} Q_\tau^2 A_{1/2}(\tau_\tau) + A_1(\tau_W), \quad X = LO, NLO, \quad (3.26)$$

and the deviation from the SM amplitude as

$$\begin{aligned} \Delta A(\gamma\gamma) = & - \sum_{q=t,b,c} \frac{4}{3} \left(\frac{\bar{c}_H}{2} + \bar{c}_q \right) 3Q_q^2 A_{1/2}^{NLO}(\tau_q) - \left(\frac{\bar{c}_H}{2} + \bar{c}_\tau \right) \frac{4}{3} Q_\tau^2 A_{1/2}(\tau_\tau) \\ & - \left(\frac{\bar{c}_H}{2} - 2\bar{c}_W \right) A_1(\tau_W). \end{aligned} \quad (3.27)$$

The decay width implemented in **eHDECAY** in the SILH case is thus the following

$$\begin{aligned} \Gamma(\gamma\gamma)|_{SILH} = & \frac{G_F \alpha_{em}^2 m_h^3}{128 \sqrt{2} \pi^3} \left\{ |A_{NLO}^{SM}(\gamma\gamma)|^2 + 2 \text{Re} \left(A_{LO}^{SM*}(\gamma\gamma) A_{ew}^{SM}(\gamma\gamma) \right) \right. \\ & \left. + 2 \text{Re} \left[A_{NLO}^{SM*}(\gamma\gamma) \left(\Delta A(\gamma\gamma) + \frac{32\pi \sin^2 \theta_W \bar{c}_\gamma}{\alpha_{em}} \right) \right] \right\}, \end{aligned} \quad (3.28)$$

where $A_{ew}^{SM}(\gamma\gamma)$ denotes the SM amplitude which comprises the NLO electroweak corrections [31, 35]. Equation (3.28) includes the leading $O(v^2/f^2)$ and $O(m_h^2/M^2)$ corrections, while it neglects terms of order $O(v^4/f^4)$. The electroweak corrections are implemented up to NLO, neglecting corrections of $O[(\alpha_2/4\pi)(v^2/f^2)]$ and of $O[(\alpha_2/4\pi)^2]$. Finally, the QCD corrections are included up to NLO, and mixed terms of $O[(\alpha_2/4\pi)(\alpha_s/4\pi)]$ are neglected.

3.4 Decay into $Z\gamma$

In the SM the Higgs boson decay into a Z boson and a photon is mediated by W boson and heavy fermion loops. Adopting the parametrization of the non-linear Lagrangian, the

⁹For a Higgs mass value of $M_H = 125 \text{ GeV}$ the running top, bottom and charm quark masses are given by $m_t = 188.03 \text{ GeV}$, $m_b = 3.44 \text{ GeV}$ and $m_c = 0.76 \text{ GeV}$. They differ from the running $\overline{\text{MS}}$ masses.

correction from the effective interaction due to the coupling $c_{Z\gamma}$ has to be considered, too, and it can become as important as the loop contributions for $(g_*^2 v^2/M^2) \approx 1$. The decay width is therefore given by (here also $\alpha_{em} \equiv \alpha_{em}(0)$):

$$\Gamma(Z\gamma)|_{NL} = \frac{G_F^2 \alpha_{em} m_W^2 m_h^3}{64\pi^4} \left(1 - \frac{m_Z^2}{m_h^2}\right)^3 \times \left| \sum_{\psi} \frac{c_{\psi} N_c Q_{\psi} \hat{v}_{\psi}}{\cos \theta_W} A_{1/2}^{Z\gamma}(\tau_{\psi}, \lambda_{\psi}) + c_W A_1^{Z\gamma}(\tau_W, \lambda_W) - \frac{4\pi}{\sqrt{\alpha_{em} \alpha_2}} c_{Z\gamma} \right|^2, \quad (3.29)$$

with $\tau_i = 4m_i^2/m_h^2$, $\lambda_i = 4m_i^2/m_Z^2$ and $\hat{v}_{\psi} = 2I_{\psi}^3 - 4Q_{\psi} \sin^2 \theta_W$ ($\psi = t, b, c, \tau$) in terms of the third component of the weak isospin I_{ψ}^3 and the electric charge Q_{ψ} . The form factors are defined by [36]

$$\begin{aligned} A_{1/2}^{Z\gamma}(\tau, \lambda) &= [I_1(\tau, \lambda) - I_2(\tau, \lambda)], \\ A_1^{Z\gamma}(\tau, \lambda) &= \cos \theta_W \left\{ 4(3 - \tan^2 \theta_W) I_2(\tau, \lambda) \right. \\ &\quad \left. + \left[\left(1 + \frac{2}{\tau}\right) \tan^2 \theta_W - \left(5 + \frac{2}{\tau}\right) \right] I_1(\tau, \lambda) \right\}. \end{aligned} \quad (3.30)$$

The functions I_1 and I_2 can be cast into the form

$$\begin{aligned} I_1(\tau, \lambda) &= \frac{\tau \lambda}{2(\tau - \lambda)} + \frac{\tau^2 \lambda^2}{2(\tau - \lambda)^2} [f(\tau) - f(\lambda)] + \frac{\tau^2 \lambda}{(\tau - \lambda)^2} [g(\tau) - g(\lambda)] \\ I_2(\tau, \lambda) &= -\frac{\tau \lambda}{2(\tau - \lambda)} [f(\tau) - f(\lambda)], \end{aligned} \quad (3.31)$$

where $f(\tau)$ is defined in Eq. (3.17) and $g(\tau)$ reads

$$g(\tau) = \begin{cases} \sqrt{\tau - 1} \arcsin \frac{1}{\sqrt{\tau}} & \tau \geq 1 \\ \frac{\sqrt{1 - \tau}}{2} \left[\ln \frac{1 + \sqrt{1 - \tau}}{1 - \sqrt{1 - \tau}} - i\pi \right] & \tau < 1. \end{cases} \quad (3.32)$$

The QCD radiative corrections [37] are small and thus have been neglected, while the NLO EW corrections are unknown. Because of the smallness of the QCD corrections, there is no relevant issue arising from the intrinsic uncertainty due to the unknown higher-order corrections, so that the choice of the scheme in which the quark masses are calculated does not play any role. In **ehDECAY** we use the pole masses for the quarks. Finally, Eq. (3.29) neglects terms further suppressed by $O(m_h^2/M^2)$, which are of higher-order in the derivative

expansion. The decay width for the MCHM4 and MCHM5 is obtained by replacing c_ψ and c_W with the coupling values of column 3 and 4 of Table 1 and setting $c_{Z\gamma} \equiv 0$.

In the SILH parametrization the decay width is computed by **eHDECAY** according to the formula

$$\begin{aligned} \Gamma(Z\gamma)|_{SILH} = & \frac{G_F^2 \alpha_{em} m_W^2 m_h^3}{64\pi^4} \left(1 - \frac{m_Z^2}{m_h^2}\right)^3 \\ & \times \left\{ |A^{SM}(Z\gamma)|^2 + 2 \operatorname{Re}(A^{SM*}(Z\gamma) \Delta A(Z\gamma)) \right. \\ & \left. + 2 \operatorname{Re} \left[-\frac{4\pi \tan \theta_W}{\sqrt{\alpha_{em} \alpha_2}} (\bar{c}_{HB} - \bar{c}_{HW} - 8\bar{c}_\gamma \sin^2 \theta_W) A^{SM*}(Z\gamma) \right] \right\}, \end{aligned} \quad (3.33)$$

where we have defined the LO SM amplitude

$$A^{SM}(Z\gamma) = \sum_\psi \frac{N_c Q_\psi \hat{v}_\psi}{\cos \theta_W} A_{1/2}^{Z\gamma}(\tau_\psi, \lambda_\psi) + A_1^{Z\gamma}(\tau_W, \lambda_W) \quad (3.34)$$

and the deviation from the SM amplitude

$$\Delta A(Z\gamma) = - \sum_\psi \left(\frac{\bar{c}_H}{2} + \bar{c}_\psi \right) \frac{N_c Q_\psi \hat{v}_\psi}{\cos \theta_W} A_{1/2}^{Z\gamma}(\tau_\psi, \lambda_\psi) - \left(\frac{\bar{c}_H}{2} - 2\bar{c}_W \right) A_1^{Z\gamma}(\tau_W, \lambda_W). \quad (3.35)$$

Equation (3.33) includes corrections of $O(v^2/f^2)$ and $O(m_h^2/M^2)$. The EW corrections are unknown, and small QCD radiative corrections have been neglected.

3.5 Decays into WW and ZZ boson pairs

The Higgs boson decay into a pair of massive vector bosons is important not only above the threshold, but also below. For example, in the SM with $m_h = 125$ GeV the branching ratio of $h \rightarrow WW$ is about 20%. In **HDECAY** various options are present to compute the partial decay widths with on-shell or off-shell bosons, controlled by the ON-SH-WZ input parameter. In **eHDECAY** we have implemented the case ON-SH-WZ=0, which includes the double off-shell decays $h \rightarrow W^*W^*, Z^*Z^*$. For this case, which is obviously the most complete as it takes into account both on-shell and off-shell contributions, the partial decay width $h \rightarrow V^*V^*$ ($V = W, Z$) can be written in the following compact form [38]:

$$\Gamma(V^*V^*) = \frac{1}{\pi^2} \int_0^{m_h^2} \frac{dQ_1^2 m_V \Gamma_V}{(Q_1^2 - m_V^2)^2 + m_V^2 \Gamma_V^2} \int_0^{(m_h - Q_1)^2} \frac{dQ_2^2 m_V \Gamma_V}{(Q_2^2 - m_V^2)^2 + m_V^2 \Gamma_V^2} \Gamma(VV), \quad (3.36)$$

where Q_1^2, Q_2^2 are the squared invariant masses of the virtual gauge bosons and m_V and Γ_V their masses and total decay widths. In the parametrization of Eq. (2.7), by defining

$$a_{VV} = c_{VV} \frac{m_h^2}{m_V^2}, \quad a_{V\partial V} = \frac{c_{V\partial V}}{2} \frac{m_h^2}{m_V^2}, \quad (3.37)$$

the squared matrix element $\Gamma(VV)$ reads

$$\begin{aligned} \Gamma(VV)|_{NL} = \Gamma^{SM}(VV) \times & \left\{ c_V^2 - 2c_V \left[\frac{a_{VV}}{2} \left(1 - \frac{Q_1^2 + Q_2^2}{m_h^2} \right) + a_{V\partial V} \frac{Q_1^2 + Q_2^2}{m_h^2} \right] \right. \\ & \left. + c_V a_{VV} \frac{\lambda(Q_1^2, Q_2^2, m_h^2) (1 - (Q_1^2 + Q_2^2)/m_h^2)}{\lambda(Q_1^2, Q_2^2, m_h^2) + 12 Q_1^2 Q_2^2 / m_h^4} \right\}, \end{aligned} \quad (3.38)$$

with [38]

$$\Gamma^{SM}(VV) = \frac{\delta_V G_F m_h^3}{16\sqrt{2}\pi} \sqrt{\lambda(Q_1^2, Q_2^2, m_h^2)} \left(\lambda(Q_1^2, Q_2^2, m_h^2) + \frac{12 Q_1^2 Q_2^2}{m_h^4} \right), \quad (3.39)$$

where $\delta_V = 2(1)$ for $V = W(Z)$ and $\lambda(x, y, z) \equiv (1 - x/z - y/z)^2 - 4xy/z^2$. The second and third term in Eq. (3.38) represent the interference between the tree-level contribution and the one from the derivative operators. They are of order $O(m_h^2/M^2)$, hence next-to-leading in the chiral expansion compared to the tree-level contribution; we have consistently neglected terms quadratic in a_{VV} and $a_{V\partial V}$, since they are of $O(m_h^4/M^4)$, which is beyond the accuracy of the effective Lagrangian (2.7). Setting $a_{VV} = a_{V\partial V} = 0$ and $c_V = \sqrt{1 - \xi}$ we obtain the decay formula for the MCHM4 and MCHM5.

In the SILH parametrization the squared matrix element $\Gamma(VV)$ implemented in **eHDECAY** reads

$$\Gamma(VV)|_{SILH} = \Gamma^{SILH}(VV) + \Gamma^{SM}(VV) \frac{2}{|A_0^{SM}|^2} \text{Re}(A_0^{*SM} A_{ew}^{SM}), \quad (3.40)$$

where A_0^{SM} denotes the SM LO amplitude and A_{ew}^{SM} is the SM amplitude which comprises the NLO EW corrections [39] (the same remark as in footnote 7 applies). Furthermore,

$$\begin{aligned} \Gamma^{SILH}(VV) = \Gamma^{SM}(VV) \times & \left\{ 1 - \bar{c}_H - 2\bar{c}_T \delta_{VZ} - 2 \left[\frac{\bar{a}_{VV}}{2} \left(1 - \frac{Q_1^2 + Q_2^2}{m_h^2} \right) + \bar{a}_{V\partial V} \frac{Q_1^2 + Q_2^2}{m_h^2} \right] \right. \\ & \left. + \bar{a}_{VV} \frac{\lambda(Q_1^2, Q_2^2, m_h^2) (1 - (Q_1^2 + Q_2^2)/m_h^2)}{\lambda(Q_1^2, Q_2^2, m_h^2) + 12 Q_1^2 Q_2^2 / m_h^4} \right\}, \end{aligned} \quad (3.41)$$

with $\delta_{VZ} = 0(1)$ for $V = W(Z)$ and where we have defined,

$$\bar{a}_{WW} = -2 \frac{m_h^2}{m_W^2} \bar{c}_{HW} \quad (3.42)$$

$$\bar{a}_{W\partial W} = -2 \frac{m_h^2}{2m_W^2} (\bar{c}_W + \bar{c}_{HW})$$

$$\bar{a}_{ZZ} = -2 \frac{m_h^2}{m_Z^2} (\bar{c}_{HW} + \bar{c}_{HB} \tan^2 \theta_W - 4\bar{c}_\gamma \tan^2 \theta_W \sin^2 \theta_W) \quad (3.43)$$

$$\bar{a}_{Z\partial Z} = -2 \frac{m_h^2}{2m_Z^2} (\bar{c}_W + \bar{c}_{HW} + (\bar{c}_B + \bar{c}_{HB}) \tan^2 \theta_W)$$

The decay width (3.40) includes terms of $O(v^2/f^2)$, $O(m_h^2/M^2)$ and $O(\alpha_2/4\pi)$, while it neglects contributions of $O(v^4/f^4)$ and $O[(\alpha_2/4\pi)^2]$. The corrections of $O[(\alpha_2/4\pi)(v^2/f^2)]$ are only partly included through the terms proportional to \bar{c}_{HW} and \bar{c}_{HB} , *cf.* [1].

4 Numerical formulas for the decay rates in the SILH Lagrangian

We display here numerically approximated formulas of the Higgs decay rates valid at linear order in the effective coefficients \bar{c}_i of the SILH Lagrangian (2.2) for $m_h = 125$ GeV. All the ratios Γ/Γ_{SM} have been computed by switching off the EW corrections, since their effect on the numerical prefactors appearing in front of the coefficients \bar{c}_i is of order $(v^2/f^2)(\alpha_2/4\pi)$ and thus beyond the accuracy of the formulas implemented in **eHDECAY**. Conversely, we have fully included the QCD corrections, as they multiply both the SM and the NP terms. The numerical results are thus the following:

$$\frac{\Gamma(\bar{\psi}\psi)}{\Gamma(\bar{\psi}\psi)_{SM}} \simeq 1 - \bar{c}_H - 2\bar{c}_\psi, \quad \text{for } \psi = \text{leptons, top-quark} . \quad (4.44)$$

The QCD corrections to the decays into charm, strange or bottom quark pairs involve two-loop diagrams with top quarks loops that are rescaled differently [20]. Taking this into

account, we have the numerical results

$$\frac{\Gamma(\bar{c}c)}{\Gamma(\bar{c}c)_{SM}} \simeq 1 - \bar{c}_H - 1.985 \bar{c}_c - 0.015 \bar{c}_t, \quad (4.45)$$

$$\frac{\Gamma(\bar{s}s)}{\Gamma(\bar{s}s)_{SM}} \simeq 1 - \bar{c}_H - 1.971 \bar{c}_s - 0.029 \bar{c}_t, \quad (4.46)$$

$$\frac{\Gamma(\bar{b}b)}{\Gamma(\bar{b}b)_{SM}} \simeq 1 - \bar{c}_H - 1.992 \bar{c}_b - 0.0085 \bar{c}_t. \quad (4.47)$$

Furthermore,

$$\frac{\Gamma(h \rightarrow W^{(*)}W^*)}{\Gamma(h \rightarrow W^{(*)}W^*)_{SM}} \simeq 1 - \bar{c}_H + 2.2 \bar{c}_W + 3.7 \bar{c}_{HW}, \quad (4.48)$$

$$\begin{aligned} \frac{\Gamma(h \rightarrow Z^{(*)}Z^*)}{\Gamma(h \rightarrow Z^{(*)}Z^*)_{SM}} &\simeq 1 - \bar{c}_H - 2\bar{c}_T + 2.0 (\bar{c}_W + \tan^2\theta_W \bar{c}_B) \\ &+ 3.0 (\bar{c}_{HW} + \tan^2\theta_W \bar{c}_{HB}) - 0.26 \bar{c}_\gamma, \end{aligned} \quad (4.49)$$

$$\begin{aligned} \frac{\Gamma(h \rightarrow Z\gamma)}{\Gamma(h \rightarrow Z\gamma)_{SM}} &\simeq 1 - \bar{c}_H + 0.12 \bar{c}_t - 5 \cdot 10^{-4} \bar{c}_c - 0.003 \bar{c}_b - 9 \cdot 10^{-5} \bar{c}_\tau \\ &+ 4.2 \bar{c}_W + 0.19 (\bar{c}_{HW} - \bar{c}_{HB} + 8 \bar{c}_\gamma \sin^2\theta_W) \frac{4\pi}{\sqrt{\alpha_2 \alpha_{em}}}, \end{aligned} \quad (4.50)$$

$$\begin{aligned} \frac{\Gamma(h \rightarrow \gamma\gamma)}{\Gamma(h \rightarrow \gamma\gamma)_{SM}} &\simeq 1 - \bar{c}_H + 0.54 \bar{c}_t - 0.003 \bar{c}_c - 0.007 \bar{c}_b - 0.007 \bar{c}_\tau \\ &+ 5.04 \bar{c}_W - 0.54 \bar{c}_\gamma \frac{4\pi}{\alpha_{em}}, \end{aligned} \quad (4.51)$$

$$\frac{\Gamma(h \rightarrow gg)}{\Gamma(h \rightarrow gg)_{SM}} \simeq 1 - \bar{c}_H - 2.12 \bar{c}_t + 0.024 \bar{c}_c + 0.1 \bar{c}_b + 22.2 \bar{c}_g \frac{4\pi}{\alpha_2}. \quad (4.52)$$

5 How to run eHDECAY: Input/Output Files

The program **eHDECAY** is self-contained, like the original code **HDECAY** on which it is based. All the new features related to the Lagrangian parametrizations proposed in this paper are encoded in the main source file, `ehdecay.f`, while other linked routines are taken over from the original version. Of course **eHDECAY**, besides calculating Higgs branching ratios and decay

widths according to the non-linear, SILH or MCHM4/5 Lagrangians, also calculates the SM and MSSM ones, exactly as **HDECAY** 5.10 does. The choice can be done through the flags **HIGGS** and **COUPVAR** set in the input file. The input file for **eHDECAY** has been called `ehdecay.in` and is based on the file `hdecay.in` of the official version 5.10, supplemented by further input values. Thus, with the flag **LAGPARAM** the user can choose between the general SILH parametrization Eq. (2.2), the model-specific parametrizations MCHM4 and MCHM5 and the general non-linear Lagrangian parametrization Eq. (2.7). Furthermore, the various related couplings can be set. The input values are explained in the following:

COUPVAR, HIGGS: If **HIGGS**=0 and **COUPVAR**=1, then the Higgs decay widths and branching ratios are calculated within the parametrization chosen by:

LAGPARAM:

- 0: Non-linear Lagrangian parametrization Eq. (2.7)
- 1: SILH parametrization Eq. (2.2)
- 2: MCHM4/5 parametrization (*cf.* Table 1)

IELW: Turn off (0) or on (1) the electroweak corrections for the SILH parametrization.¹⁰

For the non-linear Lagrangian the following parameters have to be set for the couplings of the various vertices:¹¹

¹⁰Note, that this parameter **IELW** has nothing to do with the parameter **ELWK** in the input file of **HDECAY**, where the meaning of this flag is different.

¹¹We explain them here all, although they are in part already present in the input file for **HDECAY** 5.10.

<u>CV</u> : hVV vertex, (V=W, Z)	<u>Ctau</u> : $h\tau\tau$ vertex	<u>Cmu</u> : $h\mu\mu$ vertex
<u>Ct</u> : $ht\bar{t}$ vertex	<u>Cb</u> : $hb\bar{b}$ vertex	<u>Cc</u> : $hc\bar{c}$ vertex
<u>Cs</u> : $hs\bar{s}$ vertex	<u>Cgaga</u> : coupling $c_{\gamma\gamma}$	<u>Cgg</u> : coupling c_{gg}
<u>CZga</u> : coupling $c_{Z\gamma}$	<u>CWW</u> : coupling c_{WW}	<u>CZZ</u> : coupling c_{ZZ}
<u>CWdW</u> : coupling $c_{W\partial W}$	<u>CZdZ</u> : coupling $c_{Z\partial Z}$	

In case of the SILH parametrization the input values to be set in order to calculate the various couplings are:

<u>CHbar</u> : \bar{c}_H	<u>CTbar</u> : \bar{c}_T	<u>Ctaubar</u> : \bar{c}_τ	<u>Cmubar</u> : \bar{c}_μ	<u>Ctbar</u> : \bar{c}_t
<u>Cbbar</u> : \bar{c}_b	<u>Ccbar</u> : \bar{c}_c	<u>Csbar</u> : \bar{c}_s	<u>CWbar</u> : \bar{c}_W	<u>CBbar</u> : \bar{c}_B
<u>CHWbar</u> : \bar{c}_{HW}	<u>CHBbar</u> : \bar{c}_{HB}	<u>Cgambar</u> : \bar{c}_γ	<u>Cgbar</u> : \bar{c}_g	

In the MCHM4/5 parametrization we have the input values:

FERMREPR:

- 1: MCHM4
- 2: MCHM5

XI: the value for ξ

For example:

```
COUPVAR = 1
HIGGS = 0
:
***** LAGRANGIAN 0 - chiral 1 - SILH 2 - MCHM4/5 *****
LAGPARAM = 0
**** Turn off (0) or on (1) the elw corrections for LAGPARAM = 1 or 2 ****
IELW = 1
```

***** VARIATION OF HIGGS COUPLINGS*****

CW = 1.D0
CZ = 1.D0
Ctau = 0.95D0
Cmu = 0.95D0
Ct = 0.95D0
Cb = 0.95D0
Cc = 0.95D0
Cs = 0.95D0
Cgaga = 0.005D0
Cgg = 0.001D0
CZga = 0.D0
CWW = 0.D0
CZZ = 0.D0
CWdW = 0.D0
CZdZ = 0.D0
:

computes the branching ratios for $c_V = 1$, $c_\psi = 0.95$ ($\psi = t, b, c, s, \tau, \mu$), $c_{\gamma\gamma} = 0.005$, $c_{gg} = 0.001$ and $c_{Z\gamma} = c_{WW} = c_{ZZ} = c_{W\partial W} = c_{Z\partial Z} = 0$ in the general parametrization Eq. (2.7). The output is written into the files br.eff1 and br.eff2, where the Higgs mass, branching ratios and total width are reported. For the previous example, at $m_h = 125 \text{ GeV}$ and for all the other parameters set at their standard values, the output reads

MHSM	BB	TAU TAU	MU MU	SS	CC	TT

125.000	0.5895	0.5654E-01	0.2002E-03	0.2161E-03	0.2569E-01	0.000

MHSM	GG	GAM GAM	Z GAM	WW	ZZ	WIDTH

125.000	0.9611E-01	0.1932E-03	0.1526E-02	0.2045	0.2554E-01	0.4129E-02

All the input parameters of the corresponding run are printed out in the file `br.input`. Otherwise, setting `COUPVAR=0`, the program produces the usual output files with SM or MSSM results according to the `HDECAY` 5.10 version.

6 Conclusions

We have described the Fortran code `eHDECAY`, which calculates the partial widths and the branching fractions of the decays of the Higgs boson in the Standard Model and its extension by the dimension-6 operators of the SILH Lagrangian (2.2). The program also implements the more general non-linear effective Lagrangian (2.7), which does not rely on assuming the Higgs boson to be part of an $SU(2)_L$ doublet. In the SM, all decay modes are included as in the original version of `HDECAY`. The corrections due to the effective operators have been included consistently with the multiple perturbative expansion in the number of derivatives, fields and SM couplings. The level of approximation of the formulas implemented in `eHDECAY` has been discussed in detail for each decay final state. The QCD corrections to the hadronic decays as well as the possibility of virtual intermediate states have been incorporated according to the present state of the art. The QCD corrections are assumed to factorize also in the presence of higher-dimension operators, so that they are included in factorized form in all extensions of the SM. For the SILH case we have added the electroweak corrections to the SM part only and left the dimension-6 contributions at LO in the context of electroweak corrections, since deviations from the SM case are assumed to be small. In the case of the non-linear Lagrangian however, deviations can be large so that the non-factorizing electroweak corrections to the SM part are subleading and thus have not been taken into account for consistency.

The program is fast and can be used easily. The basic SM and SILH/non-linear input parameters can be chosen from an input file. Examples of output files for the decay branching

ratios have been given.

Since electroweak corrections involving the novel operators have not been calculated yet, the treatment of this type of corrections is not complete. During the coming years one may expect that these electroweak corrections will be determined so that the existing code `eHDECAY` can be extended to incorporate them. For the moment the present version of `eHDECAY` provides the state-of-the art for the partial Higgs decay widths and branching ratios in extensions of the SM by a SILH or a non-linear effective Lagrangian.

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